BOUNDARY LAYER IN THE VICINITY OF THE STAGNATION POINT OF A BODY OF AXIAL SYMMETRY IN A TURBULENT STREAM

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The flow in the boundary layer in the vicinity of the stagnation point of a flat plate is examined. The outer stream consists of turbulent flow of the jet type, directed normally to the plate. Assumptions concerning the connection between the pulsations in velocity and temperature in the boundary layer and the average parameters chosen on the basis of experimental data made it possible to obtain an isomorphic solution of the boundary layer equations. Equations are obtained for the friction and heat transfer at the wall in the region of gradient flow taking into account the effect of the turbulence of the impinging stream. It is shown that the friction at the wall is insensitive to the turbulence of the impinging stream, while the heat transfer is significantly increased with an increase in the pulsations of the outer flow. These properties are confirmed by the results of experimental studies [1-4].

1. Fields of Velocity and Heat Content in the Boundary Layer in the Presence of Turbulent Pulsations in the Outer Stream. It follows from stability theory that the flow in the boundary layer in the vicinity of the stagnation point remains resistant to disturbances penetrating into the boundary layer because of the presence of a negative pressure gradient. The flow in the boundary layer remains laminar.

However, the pulsations of the outer turbulent stream cannot disappear instantaneously at the edge of the boundary layer and are propagated into the boundary layer, at least into its outer part. This leads to the appearance of additional stresses of friction and heat flow, due to the terms $-\rho \langle v_x' v_r' \rangle$ and $-\rho \langle v_x' h' \rangle$, where v_x' and v_r' are the components of the velocity pulsation transverse and longitudinal relative to the wall, h' is the pulsation of heat content, and ρ is the flow density.

As the results of an experimental study [5] showed, the peculiarities of flow in the boundary layer in the vicinity of the stagnation point prohibit the use of the Prandtl equation or its modifications [6-8] to find the relationships between the average and pulsation flows. The inadequacy of equations for the mixing length, which have found wide application for the calculation of turbulent boundary layers, is connected with the different nature of the formation of pulsations in an ordinary turbulent boundary layer and in a boundary layer in the vicinity of the stagnation point.

In the first case the formation of turbulence is caused by the loss of stability of the boundary layer. In the second case the formation of turbulence is connected with the penetration from the main flow of pulsations, which, because of the stability of the boundary layer, die out in it.

If the Prandtl equation is used for the longitudinal velocity pulsation

$$\sqrt{\langle v_r'^2 \rangle} = l \frac{\partial v_r}{\partial x}$$

then at the outer edge of the boundary layer with a final value of the mixing length l we obtain $\sqrt{\langle v_r'^2 \rangle}\big|_\delta = 0$, while in the real flow the root-mean-square value of the longitudinal component of the velocity pulsation has a final value equal to the root-mean-square value of the velocity pulsation in the impinging stream.

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Theoretical studies of the effect of the turbulence of the impinging stream on the flow and heat exchange in the boundary layer are conducted in [9] for plane flow and in [10] for axially symmetric flow. The equations used in these works for the turbulent viscosity ν_ε, as shown in [5], inaccurately describe the flow in the boundary layer. Therefore, the results of the calculations obtained in these works do not coincide with the results of experimental studies conducted in a broad range of variation of the parameters of the impinging stream. In the present work the solution is obtained on the basis of the experimental results of [5, 11].

In [5] the results are presented for an experimental study of the flow when an axially symmetrical jet impinges on a plate located normal to the jet axis. The velocity U_a at the nozzle mouth was varied from 7 to 30 m/sec, and Reynolds number $R_a = U_a d_a/\nu$ varied from $4 \cdot 10^4$ to $5 \cdot 10^5$, and the initial turbulence (at the nozzle mouth) was $\varepsilon_a = 0.015$. Here d_a is the diameter of the exit cross section of the nozzle and ν is the kinematic viscosity of the stream. The time average of the stream velocity and the longitudinal component of the velocity pulsations were measured in the region of interaction of the jet with the plate and in the wall boundary layer. It was established that in the wall boundary layer the flow remains resistant to the pulsations of the impinging stream. The time-averaged velocity profile coincides with the velocity profile for a laminar boundary layer in the vicinity of the stagnation point.

The results of the experimental studies presented in [5] were used to solve the equations of motion of a liquid in the boundary layer of a body of axial symmetry in the vicinity of the stagnation point, which can be written in the form

$$\begin{gathered}\frac{\partial r v_r}{\partial r} + \frac{\partial r v_x}{\partial x} = 0 \\ v_r \frac{\partial v_r}{\partial r} + v_x \frac{\partial v_r}{\partial x} = \beta^2 r + \nu \frac{\partial^2 v_r}{\partial x^2} - \frac{\partial}{\partial x}\langle v_x' v_r'\rangle \\ v_r \frac{\partial H}{\partial r} + v_x \frac{\partial H}{\partial x} = \frac{\nu}{\Pr}\frac{\partial^2 H}{\partial x^2} - \frac{\partial}{\partial x}\langle v_x' h'\rangle\end{gathered} \tag{1.1}$$

with the boundary conditions

$$v_r = v_x = 0,\ H = H_w\ (x = 0);\ v_r = \beta r,\ H = H_0\ (x = \infty)$$

Here r and x are the longitudinal and transverse coordinates relative to the plane of the plate, respectively, v is the velocity, h is the heat content, H is the total heat content, v' and h' are the pulsation components of the velocity and heat content, Pr is the Prandtl number, β is the velocity gradient at the outer edge of the boundary layer, the index w pertains to parameters at the wall, and the index 0 pertains to stagnation parameters of the impinging stream.

The system (1.1) is obtained with the following assumptions: 1) the liquid is incompressible; 2) the wall temperature is constant; 3) the thermophysical characteristics are constant; 4) the velocity distribution U_δ at the outer edge of the boundary layer obeys a linear law (β =const).

For closure of the system it is necessary to use equations relating the pulsation characteristics of the stream with the average characteristics. According to the data of [5] one can write

$$\sqrt{\langle v_r'^2\rangle} / \sqrt{\langle v_r'^2\rangle}\,|_\delta = v_r/U_\delta \tag{1.2}$$

Assuming by analogy that

$$\sqrt{\langle v_x'^2\rangle} / \sqrt{\langle v_x'^2\rangle}\,|_\delta = v_r/U_\delta, \qquad \sqrt{\langle h'^2\rangle} / \sqrt{\langle h'^2\rangle}\,|_\delta = (H - H_w)/(H_0 - H_w) \tag{1.3}$$

the turbulent fluxes of momentum and heat can be represented in the form

$$\begin{gathered}-\rho\langle v_x' v_r'\rangle = R_1 \rho (v_r/U_\delta)^2 \sqrt{\langle v_r'^2\rangle\,|_\delta}\sqrt{\langle v_x'^2\rangle\,|_\delta} \\ -\rho\langle v_x' h'\rangle = R_2 \rho \frac{v_r}{U_\delta}\frac{H - H_w}{H_0 - H_w}\sqrt{\langle v_x'^2\rangle\,|_\delta}\sqrt{\langle h'^2\rangle\,|_\delta}\end{gathered}$$

Here R_1 and R_2 are correlation coefficients which we will assume to be constants; the index δ pertains to parameters at the outer edge of the boundary layer.

Using the equations obtained, the isomorphic coordinates r and $\eta = x\sqrt{\beta/\nu}$, and the functions

$$F'(\eta) = v_r/U_\delta, \quad S(\eta) = (H - H_w)/(H_0 - H_w)$$

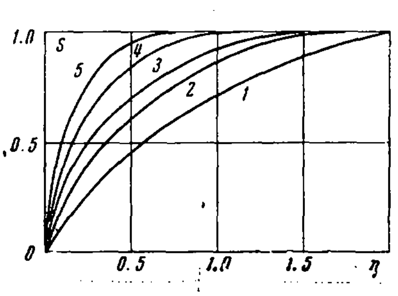

Fig. 1

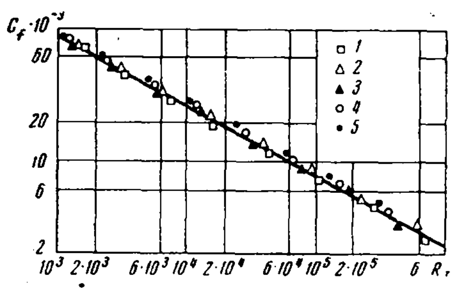

Fig. 2

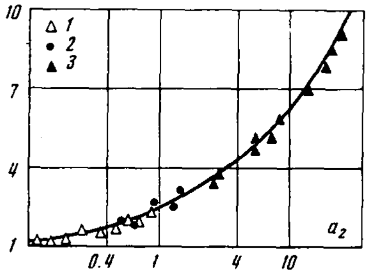

Fig. 3

the original system (1.1) is transformed to the form

$$F''' = -1 + F'^2 - 2FF'' + a_1F'F''$$
$$S'' = -\Pr(2F + a_2F')S' - a_2\Pr F''S \qquad (1.4)$$
$$a_1 = 2R_1\varepsilon^2 v_\infty^2 / r\sqrt{\beta^3\nu}, \qquad a_2 = R_2\varepsilon^2 v_\infty/\sqrt{\beta\nu}$$

with the boundary conditions

$$F = F' = S = 0 \ (\eta = 0), \qquad F' = S = 1 \ (\eta = \infty)$$

Here a_1 and a_2 are turbulence parameters, ε is the intensity of turbulence, and the index ∞ pertains to the parameters of the impinging stream.

The first equation of (1.4) is not strictly isomorphic thanks to the last term on the right. An analysis shows that when $r \geq 10\,\delta$, where δ is the thickness of the boundary layer, this term can be neglected.

The solution of system (1.4) was obtained on an electronic computer.

To analyze the effect of the nonisomorphic term of Eqs. (1.4) a solution was formally conducted with different values of a_1.

It follows from the results of the solution that in the region of gradient flow the velocity profile in the wall boundary layer is insensitive to the pulsations of the impinging stream. The weak effect of the parameter a_1 on the boundary layer makes it possible to use the solution obtained also for the small vicinity of the stagnation point. The agreement of the calculated and experimental data is satisfactory and lies within the limits of accuracy of the experimental studies.

Some results of the solution for the heat content profile (the function S) are presented in Fig. 1. Here 1, 2, 3, 4, and 5 are curves representing the results of the solution of system (1.4) at $a_2 = 0$, 1, 5, 20, and 50, respectively.

It follows from Fig. 1 that the heat content profile varies considerably with a change in the parameter a_2, i.e., the intensity of the turbulence of the main stream. The parameter a_1 exerts a weak effect on the heat content profile, hence the sensitivity of the thermal boundary layer to changes in the dynamics of the boundary layer is not great.

2. Determination of Friction and Heat Transfer. The force of friction τ_w and the heat transfer q_w at the wall are written in the form

$$\tau_w = \mu U_\delta \sqrt{\beta/\nu}\,F''(0), \qquad q_w = (\lambda/c_p)(H_0 - H_w)\sqrt{\beta/\nu}\,S'(0) \qquad (2.1)$$

where $F''(0)$ and $S'(0)$ are determined from the solution of system (1.4) presented in Part 1, and μ, λ, and c_p are the dynamic viscosity, coefficient of thermal conductivity, and heat capacity of the gas, respectively.

The local coefficient of friction is equal to

$$c_f = \tau_w / {}^1\!/_2\rho U_\delta^2 = 2.62\,R_r^{-0.5}$$
$$R_r = U_\delta r/\nu \qquad (2.2)$$

A comparison of the results of a calculation from (2.2) with the values of c_f determined from the data of [5, 11] is presented in Fig. 2. Here the calculation from (2.2) is given by a solid line, while the numbers 1-5 give the results of the experimental study with distances from the nozzle mouth to the plate of 1, 2, 3, 4, and 5 diameters of the exit cross section of the nozzle, respectively. It follows from the comparison that the turbulence of the impinging stream does not exert a significant effect on the coefficient of friction.

The variation in the dimensionless heat transfer gradient S'(0) with different Prandtl numbers ($0.5 < Pr < 1.5$) is satisfactorily approximated by the equation

$$S'(0) = 0.763\ \mathrm{Pr}^{0.4}\ (1 + 1.38\ a_2^{0.555}) \tag{2.3}$$

Equation (2.3) is the reference equation for the determination of the heat transfer from the gas to the wall in the vicinity of the stagnation point of an axially symmetrical body with allowance for the turbulence of the impinging stream. After the substitution of S'(0) into (2.1) the following equations are obtained for the heat flux and the coefficient of heat transfer, respectively:

$$\begin{aligned} q_w &= 0.763\ \mathrm{Pr}^{0.4}\ (\lambda/c_p)\ \sqrt{\beta/\nu}\,(H_0 - H_w)\ (1 + 1.38a_2^{0.555}) \\ \alpha &= 0.763\ \mathrm{Pr}^{0.4}\lambda\ \sqrt{\beta/\nu}\,(1 + 1.38a_2^{0.555}) \end{aligned} \tag{2.4}$$

The second equation of (2.4) at $a_2 = 0$ is converted into the well-known equation for α in the vicinity of the stagnation point of a body of axial symmetry in the case of a laminar boundary layer [12].

Thus,

$$\alpha/\alpha_{\varepsilon=0} = N/N_{\varepsilon=0} = 1 + 1.38a_2^{0.555}, \quad N = \alpha d_a/\lambda \tag{2.5}$$

The dependence (2.5), illustrated in Fig. 3 by a solid line, allows one to calculate the heat transfer coefficient in the vicinity of the stagnation point from the known value of the turbulence parameter a_2 of the impinging stream. The experimental data of [1, 2, 4], which are designated by the numbers 1, 2, and 3 for streamline flow around a flat plate mounted normal to the jet axis, are also presented here. For the determination of a_2 the correlation coefficient R_2 was taken as 0.4, which corresponds to the turbulent boundary layer at the flat plate. The satisfactory correspondence of the results of the calculation with the experimental data obtained in a broad range of variation of the parameters of the jet (Mach number at the nozzle mouth $M_a = 0$-3, $R_a = 10^4$-10^6, stagnation temperature $T_0 = 300$-4000°K, adiabatic index $k = 1.25$-1.4) makes it possible to use the equations presented above to calculate the friction and heat transfer during the jet interaction in the region of gradient flow.

LITERATURE CITED

1. I. P. Ginzburg, I. A. Belov, V. A. Zazimko, and V. S. Terpigor'ev, "Problem of the effect of turbulence characteristics on heat exchange of a supersonic jet with a flat barrier," in: Heat and Mass Transfer [in Russian], Vol. 1, Énergiya, Minsk (1968).
2. I. A. Belov, G. F. Gorshkov, V. S. Komarov, and V. S. Terpigor'ev, "Experimental study of heat exchange of a subsonic jet with a normally located flat barrier," Inzh.-Fiz. Zh., 20 (1971).
3. P. M. Brdlik and V. K. Savin, "Heat exchange in the vicinity of the critical point during axially symmetrical streamline flow around a flat surface located normal to the stream," Inzh.-Fiz. Zh., 10, No. 4 (1966).
4. E. H. Comfort, T. J. O'Connor, and L. A. Cass, "Heat transfer resulting from the normal impingement of a turbulent high temperature jet on an infinitely large flat plate," Proceedings of Heat Transfer and Fluid Mechanics Institute, Santa Clara, Calif. Univ. Press, Stanford, Calif. (1966).
5. I. A. Belov, G. F. Gorshkov, V. S. Komarov, and V. S. Terpigor'ev, "Experimental study of gas dynamic parameters during streamline flow around a barrier," Izv. Akad. Nauk SSSR, Mekhan. Zhidk. i Gaza, No. 2, (1971).
6. G. Schlichting, Boundary Layer Theory [Russian translation], Izd-vo Inostr. Lit., Moscow (1956).
7. S. S. Kutateladze, Boundary Turbulence [in Russian], Novosibirsk (1970), Part 1.
8. S. V. Patankar and D. B. Spalding, Heat and Mass Transfer in Boundary Layers [Russian translation], Énergiya, Moscow (1971).
9. M. C. Smith and A. M. Kuethe, "Effects of turbulence on laminar skin friction and heat transfer," Phys. Fluids, 9, No. 12 (1966).
10. I. A. Belov and V. S. Terpigor'ev, "Allowance for turbulence in calculating heat exchange at the stagnation point of a jet interacting normally with a flat barrier," Inzh.-Fiz. Zh., 17, No. 6 (1969).

11. W. Brady and G. Ludwig, "Theoretical and experimental studies of impinging uniform jets," J. Amer. Helicopter Soc., 8, No. 2 (1963).

12. M. Sibulkin, "Heat transfer near the front critical point of a body of rotation," Mechanics. Collection of Translations and Reviews of Foreign Literature [in Russian], No. 3 (1953).